Mathematical Morphology and Applications - Mid-Sem Exam

BMath(Hons.) Third Year

February 17, 2017

Instructions: There are 7 questions altogether. Marks corresponding to each question is indicated in bold. Answer as many as you can. Maximum score : **40** marks. Maximum time : **3 hrs.**

(1) Let $Y \subset \mathbb{R}$ be a fixed set and $\mathcal{P}(\mathbb{R})$ denote the power set of $\mathbb{R}$. A transformation $\Psi : \mathcal{P}(\mathbb{R}) \to \mathcal{P}(\mathbb{R})$ is defined as $\Psi(X) = X \cup Y$.

(a) Is this transformation increasing, extensive, idempotent?

(b) Does there exist a dual transformation? If so, indicate it.

[1+1+1+1]

(2) An incorrect implementation of in-place dilation w.r.t $B = \{(-1,0),(0,-1),(0,0),(0,1),(1,0)\}$ of a 2-D binary image X is given by algorithm 1 (assuming a square grid).

(a) Why does algorithm 1 yield incorrect results?

(b) Assuming that X is a $n \times n$ 2-D image, modify algorithm 1 or provide your own algorithm to yield a correct implementation which has $O(n^2)$ time complexity and uses $O(1)$ auxiliary space. You will have to justify the correctness of the algorithm along with the time and space complexities.

[3+6]

Algorithm 1 Incorrect Dilation Algorithm

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1:  procedure DILATE(X, B)                              ▷ The dilation of X by B
2:      l ← X.shape[0] and b ← X.shape[1]               ▷ Identify the length and width of the image X
3:      for i < l do                                    ▷ Iterating over all the rows
4:          for j < b do                                ▷ Iterating over all the columns
5:              if X[i][j] == 1 then                    ▷ Identifying the foreground pixels
6:                  if i > 0 then                       ▷ Lines 6-13: Checking the boundary conditions
7:                      X[i − 1][j] = 1
8:                  if j > 0 then
9:                      X[i][j − 1] = 1
10:                 if i < l − 1 then
11:                     X[i + 1][j] = 1
12:                 if j < b − 1 then
13:                     X[i][j + 1] = 1
14:     return X                                        ▷ Outputs the dilated image X
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(3) Prove that an elementary hexagon may be generated by three successive dilations of a point by three judiciously chosen segments. Then, deduce an algorithm that allows to obtain the dilation by a hexagon.

[3+3]

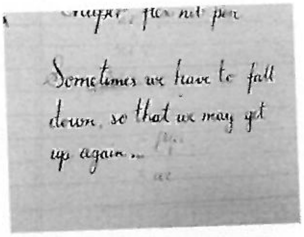

Figure 1: A smeared image (source: Quora)

(4) The notion of opening can be defined in a more general way. An algebraic opening is any transformation that is increasing, idempotent and anti-extensive. The notion of closing is defined by duality: increasing, idempotent and extensive.

(a) Is the operation consisting in extracting particles (binary case) with at least one hole an opening? Justify.

(b) Consider a family (ϕ_i) of closings (gray scale case). Prove that $\phi = \inf_i \phi_i$ is a closing.

[3+3]

(5) Suppose you are working with five operators namely Dilation, Erosion, Opening, Closing and Identity on a binary image. Assume that the same structuring element is used across all the operators.

(a) Why are the above conditions insufficient for a complete ordering (w.r.t inclusion) of these operators? Illustrate with explicit examples.

(b) Provide a sufficient condition so that they admit a complete ordering. Also, indicate the complete ordering.

[3+2+6]

(6) Suppose that you have a smeared image of a hand-written text document (see Fig 1). Which among the following procedures would enhance the quality of the document better? Explain.

- Opening
- Thinning followed by Pruning

[4]

(7) Prove that the reconstruction by geodesic dilation of the erosion of f (resp. X) of size n by the structuring element B conditionally to f (resp. X) is an algebraic opening (assume that the center of the structuring element B is a point of B).

[4]